MEASURING OF YOUNG'S MODULUS OF THIN SAMPLES USING THE QUICK BENDING VIBRATIONS OF SEARLE'S PENDULUM

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Abstract: In this paper we present accurate measurements of elastic modulus of thin quick-vibrating wire samples by Searle´s pendulum. We provide detailed statistical analysis of measurement of one "non-traditional" sample - with a rectangular cross-section. In our paper we present the measurement of Young's modulus at quick-vibrating samples where vibrations are registered and analysed by electronic sensor or camera. Also, other necessary instruments (micrometre, calibre, weight) were on an electronic basis, which was a guarantee of high accuracy measurements. The degree of an accuracy being achieved was subjected by a detailed theoretical analysis, using knowledge of theory of the uncertainties.

1 Introduction

Elastic modulus (also called tensile modulus or Young's modulus) E belongs to the most important material constants. It determines the relation between mechanical stress σ along the axis, and strain ε at axial loading, in the form $\sigma = E\varepsilon$, which is valid in the range of Hooke's law. Higher loading of the sample may result in exceeding the limits of elastic behaviour of the material.

There exist several ways for measuring this quantity. The best know methods are as follows: mechanical (static and dynamic), acoustic, ultrasonic, resonant, optical, etc. Mechanical methods are the most suitable for measuring elastic modulus E of thin samples, such as rods, wires, columns, fibres, etc. Application of the static methods (e.g. direct prolongation, two- and three- point bending etc.) however, is rather disadvantageous, as they can hardly reach accuracy better than 10 % [1].

Greater accuracy can be achieved using dynamic methods. Elastic modulus E can be determined with several per cent accuracy by means of vibrating samples at single - or three-point bending [2], or by balance of apparatus so called as Searle´s pendulum [3], [4].

In the case of pendulum device it is, however, a disadvantage that this procedure can only be used for thin samples of wire or fibre form. Additionally, using standard optical techniques (that is monitored and accounted oscillations by the eye), the area of useful samples would be further reduced to the set in which the vibration movement is relatively slow, and such the counting of oscillations is manageable "by the naked eyes".

In our paper we present the measurement of Young's modulus at quick-vibrating samples where vibrations are registered and analysed by electronic sensor or camera. Also, other necessary instruments (micrometre, calibre, weight) were on an electronic basis, which was a guarantee of high accuracy measurements. The degree of an accuracy being achieved was subjected by a detailed theoretical analysis, using knowledge of theory of the uncertainties.

2 Description of measurement

Dynamic method of measuring the modulus of elasticity with usually used bending oscillations in single- or three- point bend brings one disadvantage: the oscillations are too fast, and therefore difficult to measure.

The situation would be simpler if we should reduce their frequency somehow.

2.1 Searle's pendulum

One such method has been invented by American physicist G.F.C Searle [3], [4]. He suggested a simple idea - to attach the ends of the measuring sample to the flywheel housings. These subjects "remove from the powder" a part of their vibratory mechanical energy and thus significantly slow down the process to the level of electronically readable, sometimes even to the naked eye.

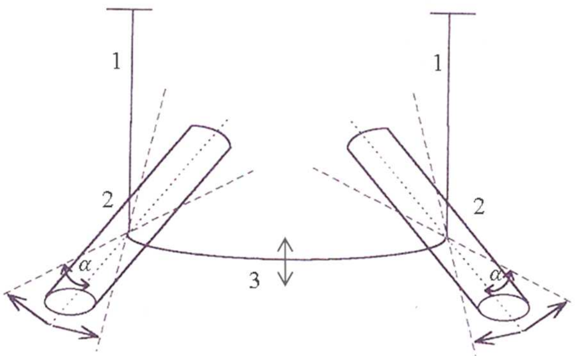

Figure 1 Searle´s pendulum.
1 – hanging threads, 2 – cylinder flywheels, 3– measured wire (arrows indicate the direction of the oscillations)

Such a device - known as Searle's pendulum - it is suitable for measuring samples with a small cross section. It consists of three main parts (Figure 1): between two hinge yarns 1 are fixed horizontally the flywheels (cylindrical or prismatic) 2, they are connected by the measured sample 3; this one basically represents the element of "coupling". Usually it is in the form of wire, but it can also be a thin rod, thread or thin prismatic tape. Symmetrical deflection of the flywheels in the horizontal direction by the angle α performs the bending oscillating movement of the sample that is reversely transmitted to the oscillating rotary motion of flywheels - and vice versa. Both parts of a pendulum – i.e. flywheels and sample - oscillate synchronously, with the same frequency and phase.

For information – such apparatus in miniature form is currently used in textile industry for examination of the elasticity of fibres, or in botany for the analogous research of plant stalks.

Dynamic analysis of the process [2], [3] gives for the measured modulus E of routinely used samples with circular cross-section, a final relationship

$$E = \frac{8\pi l J}{r^4 T^2} \qquad (1)$$

wherein *l* is the length of the sample with the radius *r*. *T* means the oscillation period of the system, and *J* is the moment of inertia of the flywheel with respect to the perpendicular axis passing through the centre (this is the same as the direction of the hinge yarns). For the less used samples with a rectangular cross-section it is [2]

$$E = \frac{24\pi^2 l J}{a^3 b T^2} \qquad (2)$$

where *a* and *b* are the width and height of the sample (this is the flywheels detained in height direction, i.e. in the direction in which bending vibrations are possible).

But now we must differentiate the type of flywheels, too. In the case of cylindrical flywheels, as well as in our picture, the moment of inertia is given by the known relationship

$$J = m\left(\frac{L^2}{12} + \frac{R^2}{4}\right) \qquad (3)$$

The parameters *R* and *L* are the diameter and length of flywheels, and *m* means their (single) mass. In the case of square flywheels it would be

$$J = \frac{1}{12} m(A^2 + B^2) \qquad (4)$$

wherein *A* and *B* are the length and the width of the prism.

2.2 The experimental assembly for measurement

We used the illustrated apparatus (Figure 2). It consist of two homogeneous steel rollers in the role of flywheels, each having a mass m = 0,72 kg, a length L = 137 mm and a radius r = 14,6 mm. Size of moment of inertia of each of them, determined from the relation (3), had a value of J = 1,15 x 10^{-3} kg.m^2. We performed measurements of several samples of wires with circular cross section and one of them with rectangular section. All the samples had the same "active" length (i.e. the distance between the points of attachment to flywheels) l = 0,295 m.

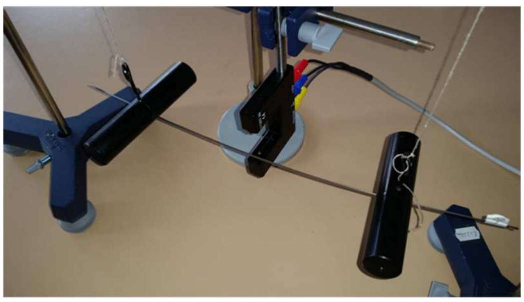

Figure 2 Experimental assembly. Vibrating wire sample crosses the infrared beam of an optical sensor (prismatic body with the shape of figure U in the centre of operation)

3 Results of measurements

Main parameters of the samples and results of measurements, including the cross-sectional geometrical dimensions, oscillation periods, together with the values of elasticity modulus are summarized below (Table 1).

Table 1 Parameters of samples and the results of measurements

Sample	Period of oscillation T (s)	Modulus of elasticity (measured) E_{meas} (GPa)	Table valued parameter E_{tab} (GPa)
Steel I – circle r = 1,00 mm	0,209	194,5	200 ± 10%
Steel II – circle r = 1,22 mm	0,138	201,5	200 ± 10%
Steel III – circle r = 1,25 mm	0,129	210,1	200 ± 10%
Copper – circle r = 1,00 mm	0,271	116,3	115 ± 10%
Aluminium – circle r = 1,00 mm	0,355	67,4	70 ± 10%
Brass – circle r = 1,70 mm	0,099	104,7	95 ± 10%
Steel – rectangular a x b = (0,79 x 2,50) mm	0,596	185,6	200 ± 10%

E modules have been obtained using equation (1) and (2). We can see that the measured values E_{meas} correspond to the table ones E_{tab}, in all cases they are lying within the corresponding intervals. (We recall a known technological fact that the existence of these relatively "wide" intervals is related to factors such as different methods of preparation, types and quantities of used ingredients, etc.). We also see that all vibrations were relatively quick - with periods of the order of tenths of a second - so that we can benefit fully the advantages our measuring apparatus, e.g. the opportunity to electronically capture the rapid oscillations.

In the next section we shall carry on a detailed analysis of the results of the steel wire sample with rarely occurring rectangular cross section with dimensions of 0,79 mm x 2,50 mm; in other words – with geometry of a sort of certain "prismatic strip". This measurement has been executed by stepwise procedure – i.e. by the method commonly used in the pendulum experiments. Finally, we had determined the percentage uncertainty in the results.

We performed measurements of 100 oscillation acts. As it is known, a stepwise procedure is the method in which we record the splits, in this case every 10 strokes.

Table 2. Results of measuring of the vibrations by stepwise procedure. The first and the third columns give the number of oscillations, the second and the fourth columns reflect the relevant times. The fifth column presents the difference of them, and corresponds to the time of 50 oscillations

i_I	$T_{i_I}(s)$	i_{II}	$T_{i_{II}}(s)$	$T_{i_{II}} - T_{i_I} = T_{i_{50}}(s)$	$\Delta T_{i_{50}} = T_{i_{50}} - \bar{T}_{50}(s)$	$(\Delta T_i)^2(s^2)$
10	5,965	60	35,790	29,825	-0,001	0,000 001
20	11,925	70	41,755	29,830	+0,004	0,000 016
30	17,895	80	47,725	29,835	+0,004	0,000 016
40	23,860	90	53,685	29,825	-0,001	0,000 001
50	29,830	100	59,650	29,820	-0,006	0,000 036
				$\bar{T}_{50} = 29{,}826$ s		$\sum(\Delta T_i)^2 = 0{,}000\ 070\ s^2$

The results will be listed in the table (Table 2) into two columns: the first one represents the first part of the measurement, i.e. range of 0 - 50 oscillations, the other corresponds to the second part from 60 to 100 strokes. We will do the differences of both of them and record it to the third column; so we get 5 different values corresponding to 50 oscillations. These results can be evaluated statistically (see last two columns + additional

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calculations). "Philosophical" principle of this method - opposite of the method of arithmetic average - lies in the fact, that the single measurement of 100 oscillations will be replaced by five 50-wobble measurements. Statistical evaluation applied here would be more precise and the resulting uncertainty would be lower, too.

We had received an average value for one period $\overline{T} = \frac{\overline{T}_{50}}{50} = 0{,}596$.

The corresponding uncertainty can be derived as in the case of physical quantity being measured by the direct route $u_T = \pm\frac{1}{50}\sqrt{\frac{\sum(\overline{T}-T_i)^2}{n(n-1)}} = 0{,}000037\ s$, wherein n = 5 means the number of measurements. This number can be considered as the size of the uncertainty of type A, too. It is clear that this value is too small for noticeable evaluating of overall uncertainty of u_E. Even if we should consider the impact of uncertainty of type B, that may be represented by the accuracy of the device in the order of five thousandths of a second, and the total time uncertainty we took as a sum of them, it will be - as we shall see later - still too little value to be strongly reflected (see calculation of overall uncertainty according to equation (5)).

We get for measuring modulus of elasticity (2)

$$E = \frac{24\pi^{2.296,3}.10^{-3}m.1{,}148.10^{-3}kg.m^2}{(0{,}79.10^{-3}m)^{3.2,50}.10^{-3}m.(0{,}593s)^2} = 185{,}64$$

.

Evaluation of the corresponding uncertainty that it will be a little lengthier. Here we must consider that the determination of the modulus of elasticity - as can be seen from equation (2) – is a function of five variables x_i; namely $E = f(l, J, a, b, T)$. In this case – in accordance with theory of measurements - the uncertainty is given by a root, containing partial derivatives with respect to all of the relevant variables and uncertainties of the following variables:

$$u_E = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial E}{\partial x_i}u_{x_i}\right)^2} =$$

$$\sqrt{\left(\frac{\partial E}{\partial l}u_l\right)^2 + \left(\frac{\partial E}{\partial J}u_J\right)^2 + \left(\frac{\partial E}{\partial a}u_a\right)^2 + \left(\frac{\partial E}{\partial b}u_b\right)^2 + \left(\frac{\partial E}{\partial T}u_T\right)^2} \quad (5)$$

The relevant partial derivatives are

$$\frac{\partial E}{\partial l} = \frac{\partial \frac{24\pi^2 lJ}{a^3bT^2}}{\partial l} = \frac{24\pi^2 J}{a^3bT^2}$$

$$\frac{\partial E}{\partial J} = \frac{\partial \frac{24\pi^2 lJ}{a^3bT^2}}{\partial J} = \frac{24\pi^2 l}{a^3bT^2}$$

$$\frac{\partial E}{\partial a} = \frac{\partial \frac{24\pi^2 lJ}{a^3bT^2}}{\partial a} = -\frac{72\pi^2 lJ}{a^4bT^2}$$

$$\frac{\partial E}{\partial b} = \frac{\partial \frac{24\pi^2 lJ}{a^3bT^2}}{\partial b} = -\frac{24\pi^2 lJ}{a^3b^2T^2}$$

$$\frac{\partial E}{\partial T} = \frac{\partial \frac{24\pi^2 lJ}{a^3bT^2}}{\partial T} = -\frac{48\pi^2 lJ}{a^3bT^3} \quad (6)$$

It is necessary to clarify the uncertainty of the moment of inertia u_J (7). This quantity has been not measured directly, but - as seen from the equation (3) – it is a function of the directly measured quantities M, L, R, and its uncertainty must therefore also be determined by means of the partial derivatives:

$$u_J = \sqrt{\left(\frac{\partial J}{\partial m}.u_m\right)^2 + \left(\frac{\partial J}{\partial L}.u_L\right)^2 + \left(\frac{\partial J}{\partial R}.u_R\right)^2},$$

e.g.

$$u_J = \sqrt{\left(\left(\frac{L^2}{12}+\frac{R^2}{4}\right)\cdot u_m\right)^2 + \left(m\frac{L}{6}\cdot u_L\right)^2 + \left(m\frac{R}{2}\cdot u_R\right)^2} \quad (7)$$

For a determination the uncertainty of time we have established an uncertainty of type B, referred above, i.e. u_t = 0,005 s. Other quantities measured by direct route (flywheel length L, its radius R and mass m, length samples l and the cross-sectional dimensions a and b) had been measured once time, only; therefore we felt the precision of measuring instruments for applying the uncertainties of them as the size of the smallest pieces on their scales. So:

u_L = 0,1 mm (sliding ruler)
u_r = 0,01 mm (micrometer)
u_l = 1 mm (ruler)
u_a, u_b = 0,01 mm (micrometer)
u_m = 1 g = 0,001 kg (laboratory scales).

Substituting into (6) gives a value of u_J = 1,5 x 10^{-5} kg.m^2. As we can see after further substituting into (5) the last three members in the roof are some orders of magnitude smaller than the previous two members, so that we can neglect them. It recognizes the value of a numerical expression of uncertainty u_E = 5,94 GPa, which represents about 3,2% against the size of the module being measured.

So, the final result can be written as E = (185,64 ± 5,94) GPa, resp. E = 185,64 GPa ± 3,2 % .

The value of total uncertainty is given by the sum of partial uncertainties of types A and B. As we know from the theory of measurement, the causes of uncertainty A are unknown. However, the causes of uncertainty B it is not difficult to determine, they related with an accuracy of instruments, uncertainty in the readings and air resistance against the vibrating motion. Other factors, such as a nonuniformity of wire thickness, directional moment of the hanging threads, heating the samples as a result of oscillations etc. are negligible.

Note: Searle's pendulum can be used - after small adjustments - as a torsion pendulum, too. It will suffice to remove the hanging threads and one of flywheels let it hang freely at the end of the measured wire. After being displaced, the flywheel will be performing the horizontal oscillations due to torsional forces of wire (Figure 3).

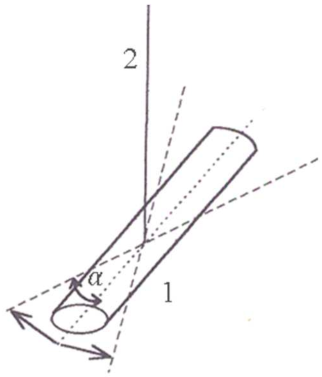

Figure 3 Flywheel (1) of Searle's pendulum hanging on wire sample (2) used as a torsion pendulum

By this way it is possible to determine the shear modulus G (8) as [4]

$$G = \frac{8\pi l J}{r^4 T_G^2} \quad (8).$$

This relationship is substantially identical with an equation (1), but the oscillation period T_G of Young's modulus in the denominator is replaced by torsion oscillation period T_G.

Using known values of E and G it is possible to determine a next important material constant – the Poisson's number μ (9) as

$$\mu = \frac{E}{2G} - 1 \quad (9).$$

The measurement of these parameters, however, has not been a filling of our work.

Conclusion

Searle's pendulum, though a simple and fairly accurate device to measure elastic modulus, is used relatively few (unfairly in our opinion). Additionally, it should be mostly in the cases of "slow oscillating" samples of circular cross section. Either as the applications in textile industry and in botany, mentioned above, is rather exceptional.

Some benefit of our paper is the fact that we performed measurements with "non-traditional" rectangular pattern. Other priority is related to higher degree of precision being achieved - because of the time was measured by an electronic detector or fast camera, respectively. This was reflected in a small error of measuring (3,2%) and in the fact that all the values.

In the near future we plan to extend the measurement on inhomogeneous samples, for example to examine how the surface coating will affect their module.

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