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## GEARING WITH VARIABLE GEAR RATIO APPLIED IN MECHANICAL SYSTEMS

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**Abstract:** The gearing with changing transmission gear ratio are used as synchronization component and for specific parameters. The gearing with changing transmission gear ratio is used in the practice, even though the "standard" gearing with constant transmission gear ratio are used more often. This article describes how to optimize the design of pitch curves of non-circular gear for given parameters. The non-circular gearing is consisting of two identical gear wheels. For a non-standard gearing was applied eccentric elliptical gear drive with continuously changing transmission gear ratio. The kinematic properties of this gearing are different from the properties of standard circular gears – spur gear. Thus, the gear ratio changes over the time of one revolution. The article is devoted to problems determining of the stress in a dangerous section of tooth foot using FEM.

### 1 Introduction

Gearboxes are one of the most used transmission mechanisms. They are based element in which machines transmit and transform mechanical energy and motion.

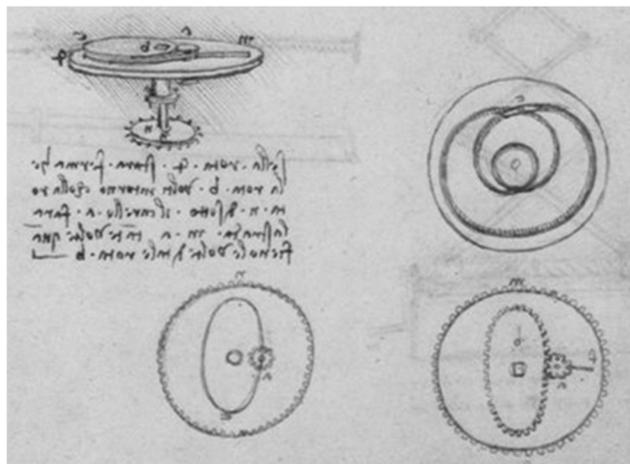


Figure 1 Sketched by Leonardo da Vinci [1]

The history of gears is probably as old as civilization itself. The earliest description of gears was written in the 4th century B.C. by Aristotle. He wrote that the "direction of rotation is reversed when one gear wheel drives another gear wheel". In practice, the most commonly used "standard" toothed gears, which can be characterized by a constant gear number and circular wheel shape. Non-circular gears are not very known, even though the idea of

non-circular gears originates from the precursors of the engineering thought. These gears were sketched by Leonardo da Vinci (Figure 1). In late XIX, century Franz Reuleaux ordered at Gustav Voigt Mechanische Werkstatt in Berlin a series of non-circular gear models to help study kinematics. The gears made at those times had simplified tooth shapes and, for this reason, the meshing conditions were not always correct.

Noncircular gears are presented as a curiosity for the gear industry history, due to their complex design and manufacturing difficulties. Nowadays, performant modelling and simulation software, advanced CNC machine tools and nonconventional manufacturing technologies enable noncircular gear design and manufacture.

As mechanisms used to generate variable motion laws, in comparison with cams, linkages, variable transmission belts, Geneva mechanisms and even electrical servomotors, noncircular gears are remarkable due to their advantages, such as the ability to produce variable speed movements in a simple, compact and reliable way, the lack of gross separation or decoupling between elements, fewer parts in the design phase, the ability to produce high strength-to-weight ratios, etc [2,3].

The applications of non-circular gears include for example textile industry machines, for improving machine kinematics resulting in the process optimization [4-6], window shade panel drives, for introducing vibration which interfere with natural oscillations and cancel them out [7,8], high torque hydraulic engines for bulkhead drives

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[9,10], mechanical presses, for optimization of work cycle kinematics. They are also using as high-power starters, mechanical systems providing progressive torque for easier starting of the machines, where progressive torque helps to overcome the start-up inertia and as forging machines, for optimizing the work cycle parameters (reducing pressure dwell time). The use of noncircular gears in industry certifies their performances, leading to new ideas for improved working conditions. Non-circular gears have their application as well oval gear flowmeter [11]. Oval gear flowmeters are categorised as positive displacement flow technology. The positive displacement flow technology allows for precise flow measurement of most clean liquids regardless of the media conductivity.

A common challenge in the design of mechanical systems is the kinematic synthesis of a mechanism in order to satisfy a set of motion characteristics [12]. Frequent requirements are to guide a rigid body through a series of specified positions and orientations (rigid body guidance), to force a coupler point to move along a prescribed trajectory (path generation), or to cause an output member to move according to a specific function of the input motion (function generation) [13,14].

The first step in the noncircular gears virtual design process is the generation of the conjugate pitch curves, starting from a predesigned law of motion for the driven element or a predesigned geometry for the driving gear pitch curve.

By designing a pair of non-circular gears, which are able to perform a proper gear ratio function, the output member of a mechanism can be effectively forced to move according to a prescribed law of motion, when operated at a constant input-velocity. This mechanism is designed to obtain a specific motion law. Detailed knowledge of meshing conditions is a prerequisite for studying kinematic conditions in gearings, as well as the strength calculation of gearing.

## 2 Non-circular gearing

Generation of this non-circular gear was by developed starting from the hypothesis such as the law of driven gear motion, variation of gear transmission ratio and design of driving gear pitch curve. This model of non-circular gear was by designed for variable transmission ratio in the range  $u = 0.25$  to  $4.0$ . This transfer should be formed by two identical wheels with the number of teeth  $z_1 = z_2 = 40$  and gearing module  $m_n = 4\text{mm}$ , the distance  $a = 160\text{mm}$  and for a one direction of rotation.

The first step in the noncircular gears design process is the generation of the pitch curves, starting from a predesigned law of motion for the driven element or a predesigned geometry for the driving gear pitch curve. For a non-standard gearing was applied eccentric elliptical gear drive with continuously changing transmission gear ratio. That is, the ellipse was used as the pitch curve (Figure 2). For the given distance, the pitch ellipse has a large half-axis  $a_e = 80\text{mm}$ , which is half of the axial distance. The

position of ellipse focus is determined by considering the desired continuously changing transmission gear ratio. For the given variable transmission ratio in the range  $u = 0.25$  to  $4.0$  is position of the ellipse focal point (the centre point  $O$  of rotation) determined by the ratio lengths  $x_1 : x_2$  equal to  $1 : 4$ . The second half-axis  $b_e = 64\text{mm}$  is determined by the distance from the focus point  $a_e = 80\text{mm}$  for the transmission ratio  $u = 1$ .

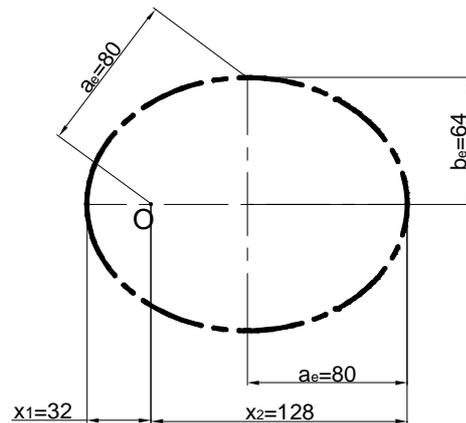


Figure 2 Pitch ellipse for gear ratio  $u = 0.25$  to  $4$

In this case, one of the conditions of a correct mesh is that the measurements of the pitch on the ellipse pitch must be kept constant. A geometric separation of the pitch ellipse into 40 identical sections (the number of teeth  $z_1 = z_2 = 40$ ) is mathematically much more difficult than in the case with the standard gear pitch circles.

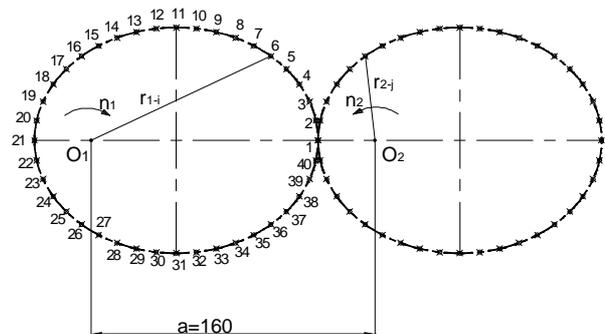


Figure 3 Designed of non-circular pitch curves

The Figure 3 shows the pitch ellipses of designed eccentric elliptical gear drive with continuously changing transmission gear ratio for a given parameters. Torque transmission ensures shape bonded between meshing gears. The gearing consists of two identical gears. The toothed number is shown for the drive wheel, for the driven wheel this numbering is the same. Wheels are designed for only one direction of rotation. The pitch ellipses must meet the condition that for each tooth the sum of the radii equal to the axial distance:

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$$r_{1-i} + r_{2-j} = a = 90 \text{ mm} \quad (1)$$

where  $r_{1-i}$  and  $r_{2-j}$  are a radius of mesh points.

### 3 Kinematic properties

In pursuit of kinematic ratios on the proposed gearings we assume from the right mesh conditions. Kinematic conditions were processed for a gear 1 (the centre of rotation at point  $O_1$ ) and the gear 2 (with the centre of rotation at point  $O_2$ ). The two gears are shown in a

kinematic dependence one graph (on the horizontal axis of the wheel teeth first).

In Figure 4 is a course of continuously changing gear ratio in one mesh generated by elliptical gear, which continuously varies in the range from  $u = 0.25$  through  $u = 1.0$  until  $u = 4.0$  and back. Thus, the gear ratio changes over the time of one revolution. A gear ratio value that is less than 1.0 signifies that this is an overdrive, and a gear ratio value greater than 1.0 signifies a speed reduction.

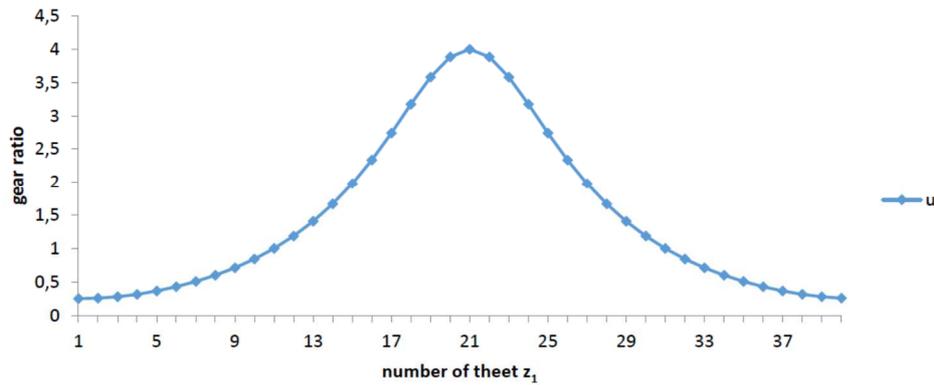


Figure 4 Gear ratio

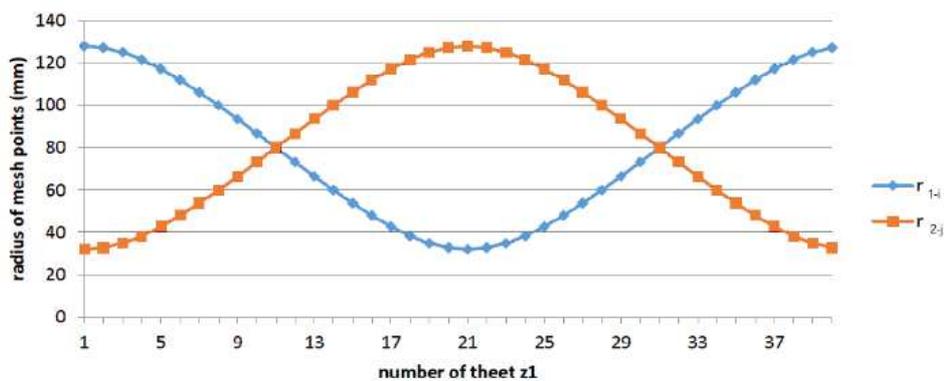


Figure 5 Radius of mesh points

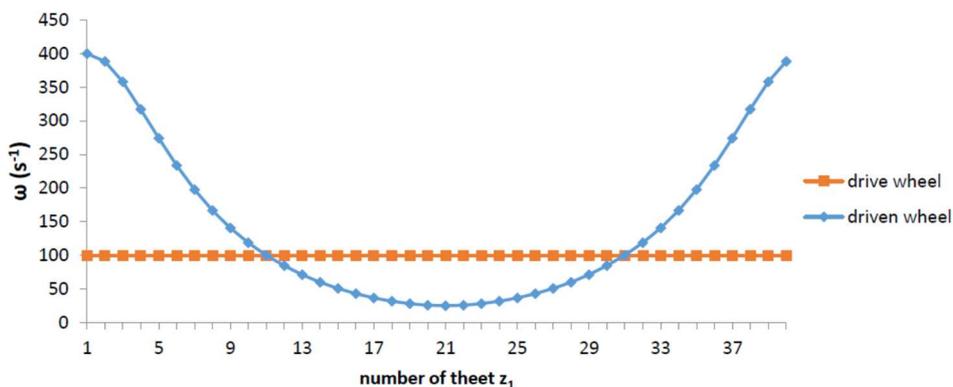


Figure 6 Rotational speed in non-circular gearing

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Figure 5 shows the progress of the meshing radii at the individual points of contact, designated as  $r_{1-i}$ , respectively.  $r_{2-i}$ , where index 1 applies to the drive wheel, index 2 for the driven wheel, index  $i$  or index  $j$  corresponds to the order number of the tooth.

The rotational speed on the drive wheel gear and the driven wheel gear is constant to standard spur gears. For designed elliptical gearing with variable transmission, the angular velocity of the driven wheel is not constant but is changed according to the continuous changing of the gear ratio. This is shown in Figure 6, if the angular velocity is on the drive wheel ( $\omega_1 = 100 \text{ s}^{-1}$ ) and the driven elliptical wheel ( $\omega_{2i}$ ).

#### 4 Stress of teeth solution by FEM

Create a geometric model of the gear is the first step to deal with tooth stress by FEM. Universal instructions to create geometry computer model does not exist [15]. The first part was to develop a functional model gear designed for the production of gears gearing for NC machine to electrospark cutting. To determine the computer model for studying deformation of the teeth using FEM was necessary to determine the material constants, define the type of finite element, and selecting appropriate boundary conditions.

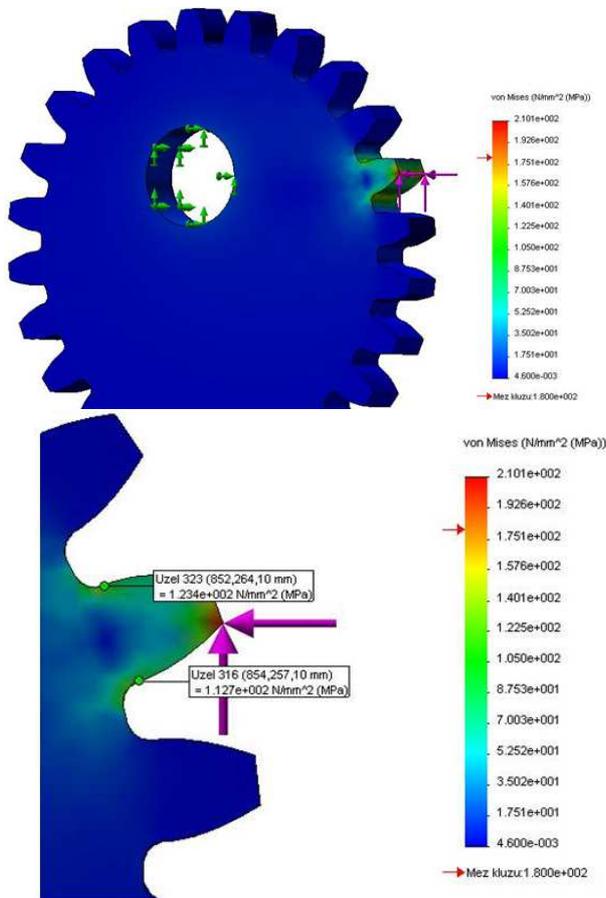


Figure 7 Sample solutions to stress in gear by FEM

The problem is solved with the gear continuously variable transmission numbers. The stress in a dangerous section of the tooth is solved using the finite element method for driving gear, the gear teeth to reach the number 0.25, 1 and 4.0.

In Figure 7 are results solutions to stress in gear by finite element method for tooth with gear ratio  $u = 1$ . Width of teeth is 10 mm, the driving torque is  $M_{k1} = 100 \text{ Nm}$ .

Because of the gear of asymmetric profile, where the teeth of one gear wheel has different shape, the stress of teeth are different.

#### 5 Conclusions

Non-circular gears are presented as a curiosity for the gear industry history, due to their complex design and manufacturing difficulties. Nowadays, performant modelling and simulation software, advanced CNC machine tools and nonconventional manufacturing technologies enable noncircular gear design and manufacture to be more feasible.

The main objective of this paper was to defined base kinematic properties of non-circular gear. Gearing was designed to meet continuous change of gear ratio during one rotation. The gearing consists of two identical gears and the basic shape of the gear wheel is formed by an ellipse. Wheels are designed for only one direction of rotation and the centre of rotation is one of the foci of ellipse. It is the gearing with variable transmission. Properties of this gearing are different from the properties of standard circular gears – spur gear. Thus, the gear ratio changes over the time of one revolution. A gear ratio value that is less than 1.0 signifies that this is an overdrive, and a gear ratio value greater than 1.0 signifies a speed reduction. For designed elliptical gearing with variable transmission, the angular velocity of the driven wheel is not constant but is changed according to the continuous changing of the gear ratio. The stress in a dangerous section of tooth calculation by standard is provided according to specific conditions. This calculation is not suitable for elliptical spur gear with variable gear ratio. The theoretical determination of the stress in the teeth is difficult for complex-shaped teeth. One way to determine the stress in a dangerous section of the tooth is a solution to this problem using the finite element method.

This elliptical gearing was by used in the drive mechanism for a new press for example. The new press kinematics result in a reduced pressure dwell time in comparison with a conventional press kinematic.

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