COMPUTER SIMULATION OF THE MECHANICAL SYSTEM WITH ONE DEGREE OF FREEDOM
Darina Hroncová; Alexander Gmiterko; Tomáš Lipták; Mojmír Bruchánek

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Abstract: This paper deals with analyze the model of the oscillating mechanical system with one degree of freedom. In this article the vibration of the car seat with driver is solved. It is a problem that is common in technical practice in the assessment of driver comfort when driving a car. It presents the possibilities of physical modeling in Matlab/Simulink. When used, it is necessary to describe the studied system with dynamic equations. The task of the paper is to create a mathematical model and computer simulation model. The results of these solutions are created in the form of graphs.

1 Introduction

Modeling of mechanical systems using available software tools is a frequently occurring phenomenon in engineering practice. Compilation of the models, systems, whether mathematical or dynamical expressed by mathematical modeling and experimental identification is the current issues in the engineering industry. Such simulated respectively modeled situations mechanical systems we can examine in Matlab and Simulink. In modeling and simulating computer may change the parameters systems conduct without any risk disrepair [1].

2 Modeling of the objects

Object modeling modeling process is a system where certain criteria assigns physical or abstract model. There are two types of models, original and abstract. In the original model can not perform experiments, as with abstract models, where we simulate real situations. We create simulations with the help of mathematical models and solution is done using numerical methods with differential equations in simulation programs.

Abstract model we can compare with the original model, but three types of similarities are known, which represent the characteristics of the model:

1. physical similarity - the same substance of the system and of the processes and also of the parameters and state space parameters,

2. mathematical similarity - is the mathematical description of systems and processes,

3. cybernetics similarity - mathematical similarity in the behavior of the internal systems. System information in the internal structure and internal variables are unknown to us [1].

3 External description of the dynamical systems

In engineering practice we can meet the following forms of external description of linear dynamic systems [2]:

1. Differential equation
2. The Laplace transform
3. Frequency response
4. Impulse response
5. Transfer characteristics
6. Nyquist response
7. Bode characteristics

4 Modeling of mechanical systems

In studying properties in mechanical systems always we replace true system with the simulation model that has the same properties. In examining of the dynamic phenomena are replacing real system with dynamic model. For this then we compiled mathematical model. In addressing vibration of mechanical systems,

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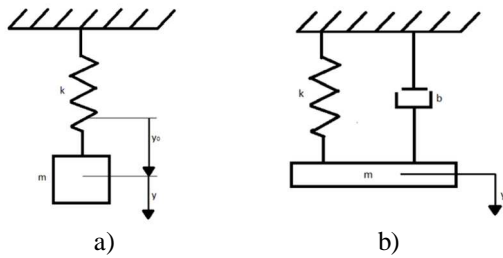constructed a mathematical model is describes with the system of differential equations [6].

Figure 1 Mechanical system a) mass and spring, b)mass, spring and damper

In the initial phase of the analysis it is appropriate as first used simple models of mechanical systems that analyze the basic properties. Basic models can be used to describe the behaviour of different pieces of equipment. There utilize for it the simple mechanical systems with one degree of freedom of movement. Types of simple vibration patterns in different types of drive power are shown in Figure 1 to Figure 3 [1], [2].
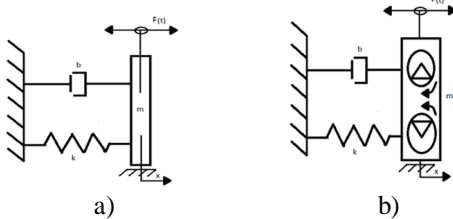

Figure 2 Mechanical system a) excitation force, b)harmonic excitation

In the following parts the article pays attention derivation to derive the dynamical equations to solve the mechanical system and its solution using Matlab and Simulink [4-5]. Use them to derive dynamical equations using Newton's equations, the method of release.
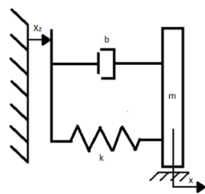

Figure 3 Mechanical system with kinematic excitation

The next part contains a model with one degree of freedom, which is the car seat with driver and his vibration.

5   Mechanical system

Anatomic car seat (Figure 4) must meet the exacting requirements that are placed on the health, comfort, convenience and safety. The emphasis is on utility ergonomic and aesthetic qualities and the ability of damping. The car seat is equipped with displacement and uprise, increased lateral edge opera guarantees a relaxed and comfortable seating [2].

In order to derive analytical mathematical model of car seat we can be based on a simplified model. This is essentially a mass m consist mass of driver m_v, car seat and all devices for damping resilient suspension with mass m_s [3].

This mechanical system is a mass m mounted on the frame with springs k and dampers b and load by the gravitational force of the driver (Figure 4c). It is the mechanical system with one degree of freedom of movement. The equation of motion is expressed by linear differential equations with constant coefficients with the right side (1). After the creating of a dynamic and mathematical model it used to simulate the program Matlab/Simulink [6], [7].
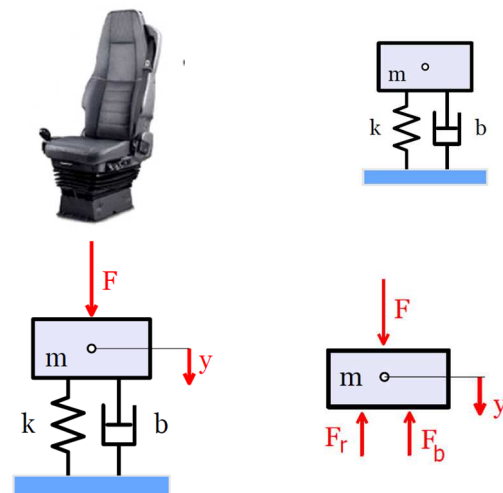

Figure 4 Mechanical system a) of car seat, b) model of the car seat, c) model with external excitation, d) model with the reaction forces and external excitation

Solving the problem of system is considered with the weight of car seat 20 kg and weight force of the driver 100 kg . The weight of the car seat with the driver is then m=120 kg. It is envisaged acceleration of gravity $g = 9.81$ m.s^{-2}, the spring constant k=9600 N/m and damping coefficient b=2400N.s/m. It is considered the excitation force that represents the gravitational force of the driver of vehicle F=m_v.g = 981N [3]. The system consists of the mass m, which is mounted on the frame with the spring with spring constant k and damper with the damping factor b (Figure 4b). In the next picture the model is loaded with constant force F and displacement of the system (Figure 4c) is y(t). Compile equations of motion is made using Newton's laws of motion. The seat is released from the bonds and the bonds are replaced by reactions as shown in Figure 4d.

Dynamical equation of the mechanical system is shown in form:

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$$m\,\ddot{y} + b\,\dot{y} + k\,y = F \qquad (1)$$

where
m – mass of the car seat and driver [kg],
F – excitation force of driver [N],
k – spring constant [N/m],
b – dumping constant [N/(m/s)].

6   Numerical solution of differential equations in Matlab

Derived equation of motion in this chapter adjusted to the system of two differential equations of first order [6]. These can also be resolved, for example, using the Matlab [14]. Equations are entered in m-files for values: m=120 kg, k=9600 N/m, b=2400 N.s/m, F=981 N.

Marking:

$$x(1) = y(t), \qquad (2)$$

$$x(2) = \frac{d\,y(t)}{d\,t}, \qquad (3)$$

we write the two first order differential equations in the form:

$$\frac{d\,x(1)}{d\,t} = x(2), \qquad (4)$$

$$\frac{d\,x(2)}{d\,t} = -\frac{b}{m}x(2) - \frac{k}{m}x(1) + \frac{F}{m} \qquad (5)$$

M-file in Matlab simulation [7]:
```
% car seat and driver
[t,x]=ode45(@funkcia_seat,[0 5],[0;0]);
% [0 5] - Time Simulation ,
% [0;0] - Initial Condition,
plot(t,x)
title('Displacement and velocity  y(t), ...
v(t)');
xlabel('Time (sec)');
ylabel('y(t) , v(t)');
grid on;
legend('y=y(t)','v=v(t)')
```
 Function is in form:
```
function dx=funkcia_seat(t,x)
m=120; k=9600; b=2400;
F=981;
dx=[x(2); -k * x(1) / m - b * x(2) / m...
+ F/m];
```
The results of the simulation are displacement y(t) and velocity v(t) of car seat with driver (Figure 5).
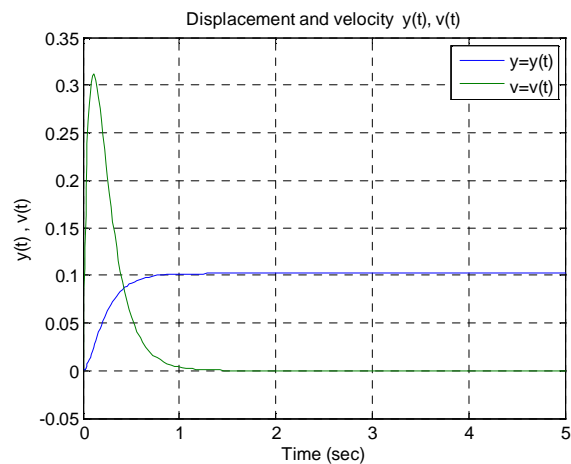

Figure 5 Displacement and velocity of car seat

Dynamic system with car seat vibration is according to the profile displacement and velocity stable [15]. After touch down on the car seat there is a transient, who is the ultimate time practically ceased. After a reasonable period of time will be displacement of car seat y = 981/9600 = 0.102 m.

7   Computer simulation in Simulink
7.1   Differential equation of the mechanical system

Using Simulink [10] in this chapter shows how to compile block diagrams by blocks located in different libraries Simulink, which is one of the best known and most widely used Matlab extensions. It uses Matlab [11] for modeling, simulation and analysis in an easy graphical user interface. Equations in the form:

$$\ddot{y} = -\frac{b}{m}\,\dot{y} - \frac{k}{m}\,y + \frac{1}{m}\,F \qquad (6)$$

is used in the simulation program Matlab/Simulink [12], and from there is constructed a further block diagram of the picture(Figure 6).
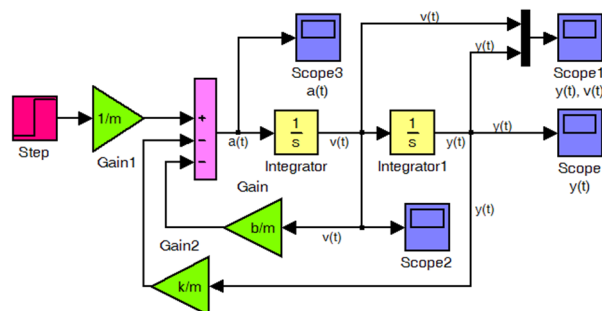

Figure 6 Block scheme of the diffential equation (6) in Simulink

From equation (4) and (5):

$$\frac{d\,y(t)}{d\,t} = v(t), \qquad (7)$$

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$$\frac{dv(t)}{dt} = -\frac{b}{m}v(t) - \frac{k}{m}y(t) + \frac{1}{m}F , \qquad (8)$$

is compile the following block diagram (Figure 7). The results of the solutions are the same as we expected (Figure5).
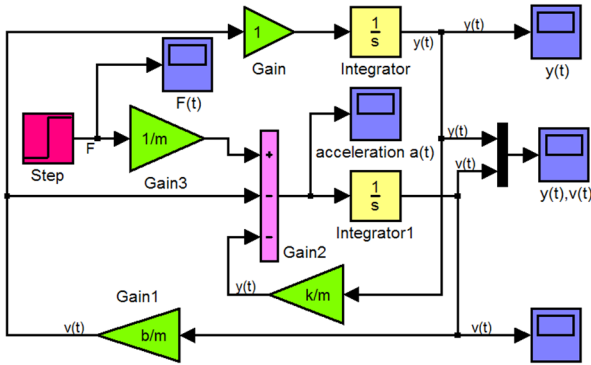

Figure 7 Block scheme of the differential equation (7)and (8)

Time course of kinematics variables of car seat and driver are displacement (Figure 8), velocity (Figure 9) and acceleration (Figure 10) in graphical form [6], [7] shown below:
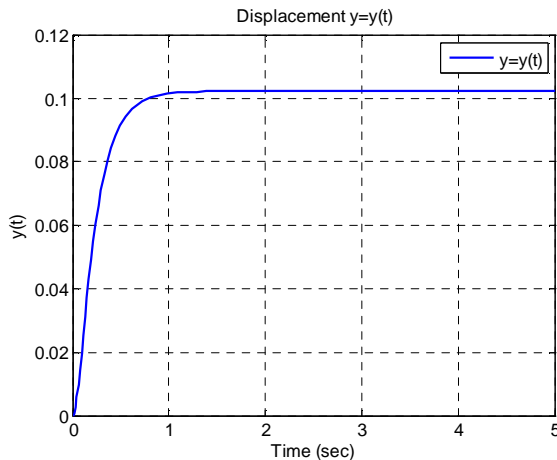

Figure 8 Displacement of the mechanical system

M-file in form:
```
figure(1)
set(1,'Name','Displacement y=y(t)')
plot(Scope_y.time,Scope_y.signals.values ...
(:,1:1),'b','LineWidth', 1.5)
title(' Displacement y=y(t)'),...
legend('y=y(t)'),
xlabel('Time (sec)'),...
ylabel('y(t)'),...
grid on
```
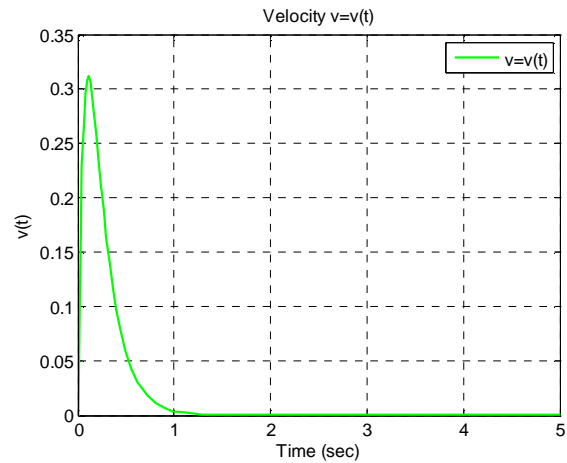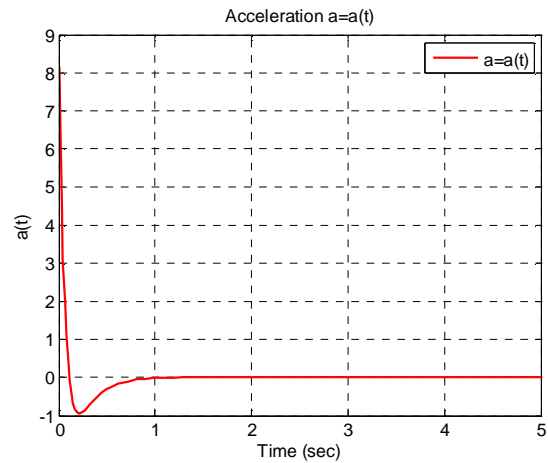

Figure 9 Velocity of the mechanical system

Figure 10 Acceleration of the mechanical system

7.2 Transfer function of mechanical system

Transfer function of the system is defined in form:

$$G(s) = \frac{Y(s)}{U(s)} \qquad (9)$$

where

Y(s) – is Laplace s-domain of the time domain function y(t), the output of system,

U(s) – is Laplace s-domain of the time domain function F(t), the input force of system.

For initial condition in form y(0)=0, v(0)=0, we write the equation of motion (1) in next form [9]:

$$mY(s)s^2 + bY(s)s + kY(s) = U(s) \qquad (10)$$

Transfer function is shown in form:

$$G(s) = \frac{1/m}{s^2 + (b/m)s + (k/m)} \qquad (11)$$

After compile the block diagram in Simulink (Figure 11) is the result of the solution of mechanical system with using the transfer function of the system (Transfer Fcn) the output y(t), which is the displacement of the mechanical system [14] ,[15].

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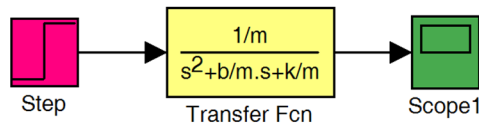Darina Hroncová; Alexander Gmiterko; Tomáš Lipták; Mojmír Bruchánek
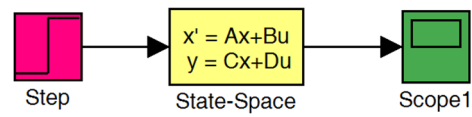

Figure 11 Simulation scheme with Transfer Fcn.

Results of the simulation with block scheme in Simulink are identical with the course in Figure 8.

7.3   State - space of the mechanical system

State–space equation of the mechanical system [7-8] in matrix form we write:

$$\begin{bmatrix} \dfrac{d\,x(t)}{dt} \\ \dfrac{d\,v(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{b}{m} \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} \cdot F(t) \quad (12)$$

The most general state-space representation of a linear system is written in the following form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}.\mathbf{x}(t) + \mathbf{B}.\mathbf{u}(t) \quad (13)$$

$$\mathbf{y}(t) = \mathbf{C}.\mathbf{x}(t) + \mathbf{D}.\mathbf{u}(t) \quad (14)$$

can be written as:

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \mathbf{A}.\begin{bmatrix} x \\ v \end{bmatrix} + \mathbf{B}.F(t) \quad (15)$$

$$y = \mathbf{C}.\begin{bmatrix} x \\ v \end{bmatrix} + \mathbf{D}.F(t) \quad (16)$$

The state space matrix are in the form:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{b}{m} \end{bmatrix} , \; \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} , \quad (17)$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} , \; \mathbf{D} = [0] , \quad (18)$$

writen as:

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{b}{m} \end{bmatrix} \cdot \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} . F(t) \quad (19)$$

where
input u(t) is the excitation force F(t) and output y(t) is the displacement,
u(t) - is called the input (or control) vector,
y(t) – is called the output vector,
x(t) – is called the state vector,
A - is the state (or space) matrix,
B – is the input matrix,
C – is the output matrix,
D - is the feedforward matrix (for many physical systems is the null matrix).
The scheme in Simulink:

Figure 12 Block scheme with State Space

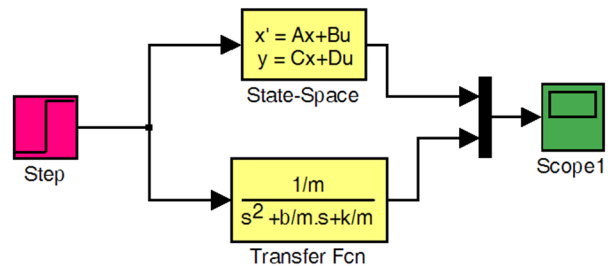For the comparison of the results is given in the following scheme:

Figure 13 Simulink scheme with State Space and Transfer Fcn

Write in the command line is rendered characteristics of the transfer function, which is displacement of the model seat of car [3]. The matrix B is multiplied by the input, in dealing with a case, force u = F:
<<step (A,u*B,C,D)
The output of the solution is the characteristics of the transfer function, which is deflection of seat [15]:
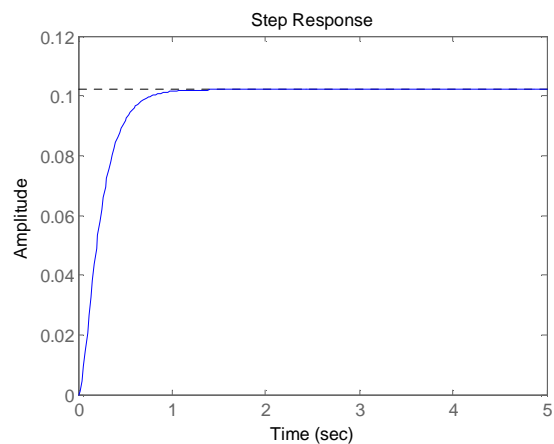

Figure 14 Step response in Simulink

Type in the command line is defined the transfer function of the system in Matlab with transformation from State-Space [12-13]:
>> [num,den] = ss2tf(A,B,C,D)
Answer from Matlab:
num =
0 0 0.0083
den =
1 20 80
Type in the command line is defined the characteristics of transfer function Step Response of the system. Numerator multiplied with inputs u, in solving

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case with force F = 981N and we define simulation time T=5 seconds:

>> step(u*num,den,T)

A window with characteristics of transfer function opens. It is identical with the course in Figure 14.

Conclusions

In this paper they were shown to solve the dynamic equations of motion of a mechanical system with one degree of freedom of the oscillating system car seat with driver. The system was calculated using computer simulation. From assembled block diagrams in Simulink program, compiled m-files in Matlab and 2D graphics are acquired waveforms kinematics variables of the system car seat with driver. The results obtained are illustrated in the number of outputs in the form of graphs and were used various block diagrams investigating mechanical system. Simulation models are compiled in different ways, using the full range of the blocks includes a library of Matlab and Simulink. The result is more detailed and deeper acquaintance with that issue. The processed results can be utilized in further addressing similar problems in mechanics and mechatronics. Article in its form is a powerful draw lessons for teaching purposes in subjects with a focus on solving dynamic systems in subject mechanics and mechatronics.

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